I. INTRODUCTION

The modeling of data as the product of two lower-dimensional (often matrix-valued) latent variables or factors has been widely used in applications as varied as psychometrics, finance, recommender systems, DNA microarray analysis, and foreground/background video separation. In finance, the factors often correspond to market segments and market styles; in recommender systems, the factors correspond to users and items. In many cases, the data matrices under investigation are dynamic, i.e., are functions of time.

Matrix factorization methods have a long history in statistics and signal processing, but most methods deal with the case where either both factors are not time-varying or one of the factors is not time-varying. In contrast, we formulate an approach for factoring dynamic matrices in which both factors are not time-varying. The forward model could simply be the identity operation, but we want to capture sudden changes unexplained by the forward model and a source of potential difficulty in optimization is the nonlinearity of the investment universe when modeling risk and return. For time series of stock prices, $S$, in the tens of thousands, $T$ in the hundreds, and $K$ thirty to fifty. For fixed income yield curves, often $K = 3$ with intuitive interpretations of the three factors as level-shift, slope, and curvature changes.

Oftentimes in these applications, the matrices that are factored are static; doing so allows the analyst to produce stable factors, but such a model cannot react quickly to changes in the market. Exponential smoothing methods with fast decay can allow dynamics in the model but result in factors highly sensitive to short-lived spikes in volatility and other market anomalies. Our proposed method, since it incorporates robust Kalman smoothing, is able to both react to changes and ignore short-lived anomalies. Parametric and semiparametric methods for bond yield curves are not as flexible and rich as our proposed approach [2], [8], [9].

In practice, we have applied the proposed approach to yield curve modeling. For example, Fig. 1 shows the yield curve for eurodollar contracts from 1998 to 2010. Factors estimated using the Kalman smoothing for different example days with $K = 3$ are shown in Fig. 2. The results show that the factors do tend to have the level-shift, slope, and curvature interpretations and vary smoothly over time, thereby reacting to market changes without sensitivity to spikes.

IV. DISCUSSION

In the big data era, modeling of high-dimensional dynamic matrices is increasingly important. We have proposed a new, efficient, and robust method for dynamic matrix factorization based on an optimization viewpoint of nonlinear Kalman smoothing that permits both factors to be time-varying. An extension of the proposed approach with orthogonality constraints on the inferred factors allows us to tackle the subspace estimation and tracking problem [10]–[12].
Fig. 1. Yield curve data for eurodollar contracts from 1998 to 2010. The curve for a given time represents the prices for the contracts at different available durations.

Fig. 2. Yield curve factor estimates using proposed Kalman smoothing method for four different example days. The three factors per plot have an intuitive interpretation as level-shift (flat curve), slope (mostly decreasing curve), and curvature. The factors track changes smoothly across time due to the robust Kalman smoothing formulation.

REFERENCES


